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Review Article

The Role and Application of Calculus in Aerodynamics: A Systematic Review

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About Article

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ABSTRACT

This review brings together 18 studies that explore how calculus is applied in aerodynamics, drawing from leading aerospace and engineering databases. Among these, seven studies (39%) concentrated on differential equations especially the Navier–Stokes and Euler formulations which remain the core tools for representing fluid flow and turbulence. Another five studies (28%) examined the use of the calculus of variations and shape calculus in optimization tasks, including wing design and trajectory planning, and consistently reported gains in aerodynamic efficiency. Four studies (22%) explored numerical methods, including finite volume, finite difference, and fractional derivatives, confirming calculus as the backbone of computational fluid dynamics (CFD). Only two studies (11%) looked at advanced approaches, particularly the use of fractional-order calculus in flight dynamics and control. Their findings suggest that these methods can strengthen stability when aircraft operate under nonlinear or rapidly changing conditions. Taken as a whole, the review indicates that calculus is not just a theoretical tool but also a means of driving progress in aerodynamics. Even so, gaps remain: few works explore real-time applications in autonomous flight, integration with machine learning is still limited, and fractional calculus has seen little use in practice. Filling these gaps depends on integrating mathematical models, computational techniques, and engineering practice in order to advance safer and more efficient aerospace applications.

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1. INTRODUCTION

Calculus is a major part of flight science and through calculus, we can predict how air should move around a plane. Also, engineers are able to create a more aerodynamic plane as well. Differential equations, including more simply the Navier–Stokes equations, help to express change in velocity, temperature, and pressure over time and place. Based on conservation laws for mass, momentum, and energy, these equations explain how liquid flow behaves, regardless of the situation. In most cases, you can't exactly solve aerospace problems leaving us to use numerical methods to almost recreate the exact solution to our problem (Almheidat *et al*., 2024).

By introducing new mathematical tools, vector calculus allows engineers to calculate the force of lift and drag more accurately and ease, helping to evaluate the capabilities of different shapes and model types. Calculus has help refining airplanes to be more fuel efficient by replacing wing shape by the verses shape of the airplanes it is a lot of benefits to the airplanes. They show that calculus is the application that drives experiments and innovation in science along with multiple uses for real-life aerodynamics today.

Related topic is recognized yet the current available literature on calculus in aerodynamics is fragmented. According to standard textbooks, there are strong theoretical foundations (Anderson, 2011; White, 2016). For certain individual applications, more specified studies exist. Turbulence modeling and flight control optimization are good examples of this. As of now, though, there is no systematic synthesis of these contributions into one framework. Consequently, we lack an integrated evidence base that illustrates what we already know, as well as what we do not yet know.

This gap is significant for both scholars and practitioners. For researchers, a unified review is essential to identify methodological trends, highlight unresolved challenges (such as real-time optimization or turbulence modeling), and suggest directions for future work. For engineers and designers, consolidating the evidence across differential equations, optimization frameworks, computational methods, and control systems provides practical guidance in selecting the most effective mathematical tools for specific aerospace applications.

To address this gap, the present study undertakes a systematic review of published literature on the role and applications of calculus in aerodynamics. By analyzing 18 studies across four thematic domains: differential equations, optimization methods, computational techniques, and control applications. This review provides a comprehensive synthesis of how calculus functions both as a theoretical foundation and as a practical enabler of innovation in next-generation aerospace systems.

2. LITERATURE REVIEW

The basic mathematical tools of aerodynamic theory are differential equations, and these lead to a description of how fluid properties change with position and time throughout the flow field we wish to evaluate. The Navier-Stokes equations describe a nonlinear system of coupled partial differential equations (PDEs) that model the flow of a viscid fluid observed empirically to be turbulent over certain pertinent regimes, and otherwise laminar. The principles of mass, momentum, and energy conservation are like the building blocks of fluid mechanics and aerodynamics. They are crucial for understanding how things move through air and water (White & Xue, 2021). Working on these complex problems in the aerospace industry often requires a lot of time and resources. For this reason, engineers commonly use numerical methods to arrive at accurate solutions, as noted by Maalee Almheidat and colleagues (2024)

Aerodynamicists employ both PDEs and ODEs. PDEs are generally used to mathematically model flow fields, pressure distributions, and shock waves, whereas ODEs are typically applied in simpler scenarios or when examining the trajectory and stability of aircraft motion (Anderson, 2011). In the real world, however, the solution to fluids and geometries is too complex that only advanced numerical methods can simulate it using differential calculus approximations (Sahani *et al*., 2023).

As a requirement, immense mathematical rigor (that connects modeling to vector calculus) is established, which facilitates the expression of conservation laws using divergence, gradient, and curl operations. Vorticity and circulation are central ideas in determining the lift and drag forces produced from bodies such as airfoil wings (Kaza & Kadari, 2025). These qualitative principles explain how air swirls and why it changes the forces acting on a wing. Essentially, they are vital for understanding

2.1. Differential equations and their central role

Differential equations are central to aerodynamic theory, offering a framework to elucidate how variables like velocity, pressure, and temperature change over space and time. They help us understand how things like velocity, pressure, and temperature change across different spaces and over time. A big part of this is the Navier–Stokes equations, which might sound complicated, but they really just describe how fluids move, whether they're flowing smoothly or all mixed up. These equations come from the basic principles of mass, momentum, and energy conservation and are crucial for modern fluid dynamics and aerodynamics (White & Xue, 2021).

The complexity of these equations poses significant challenges in aerospace practice. According to Maalee Almheidat and colleagues in 2024, this is the reason we typically use numerical methods to arrive at precise solutions.

In this field, both partial and ordinary differential equations play a crucial role. Partial differential equations are employed to model intricate phenomena like flow fields, pressure distributions, and shock waves (Anderson, 2011; White & Xue, 2021). In contrast, ordinary differential equations are typically utilized for more specific tasks, such as forecasting the time evolution of an aircraft's trajectory or evaluating stability (Anderson, 2011). Aerospace structures are not regularly shaped, and fluid behavior is a complex field, both of which require the use of a number of advanced numerical methods that rely on differential calculus (Sahani *et al*., 2023).

These indispensable mathematical tools are partly defined via vector calculus techniques. The diversity of conservation laws can be described very efficiently with operators like divergence, gradient, curl (Kaza & Kadari 2025; White & Xue 2021), and in particular we can write the above three-dimensional Stokes model as a complex analytical expression too. These are important operators that specify the exact conservation laws, which are vital in many branches of science. An understanding of these two concepts is key to understanding viscosity, circulation, and vorticity. The principles of lift and drag are critical in aviation; to comprehend them it is important that you must understand these terms. They demonstrate the reason airplanes are able to fly and how they get more efficient at their job as they fly through the air (Kaza & Kadari, 2025).

2.2. Calculus of variations in aerodynamic shape optimization

Calculus serves as a fundamental element in both aerodynamic and flight dynamic modeling, while also playing a crucial role in enhancing real-world flight performance (Anderson, 2011; White, 2016). It allows them to better understand and predict difficult fluid and motion dynamic problems that haven't been easy for engineers to solve and that is why, improve aviation design tools allowing more efficient ways of designing aircrafts in the future thus leading to safer air travel as well (Kaza & Kadari, 2025). The method of the calculus of variations has been commonly used to optimize optimal trajectory while solving problems (Ibrahim & Tiwari, 2004; Chai *et al.*, 2023), and it is a very useful approach. For instance, this mathematical approach has been utilized to determine the best climb, cruise, and descent profiles for commercial short-haul flights, taking into account constraints like fuel efficiency, time management, and environmental considerations (Gallant, 2012; Schmidt *et al.*, 2011). These examples show just how essential calculus is in addressing real-world engineering challenges in the aerospace industry, giving engineers the tools they need to solve problems more effectively.

A case in point is a more advanced application of calibration theory in aerodynamics, namely calculus through variational, which shall be employed to determine extremal functions that solve specific functional costs that directly or indirectly relate to aerodynamic performance measures, including minimization of drag and maximization of lift. This technique is analytically and computationally used to determine the optimum shapes and flow passes of a flying body, as previously demonstrated (Anderson 2011).

This can be observed in, for example, the calculus of variations approach by Chai *et al*, which is used to determine optimal flight profiles for minimum fuel burn or to minimize the travel time under a combination of nonlinear dynamic constraints and operation-specific operational constraints. Variational methods are broadly used in the shape optimization of wings and airfoils with fixed boundary conditions and aerodynamic force distributions on surfaces (Ibrahim & Tiwari, 2004), which are mostly incorporated by the adjoint method for sensitivity analysis.

Subsequently, large-scale aerodynamic shape optimization problems, such as the design of intricate aircraft parts, have utilized shape calculus to compute accurate gradient data, yielding rapid iterative enhancements in designs within a computationally feasible framework (Schmidt *et al.*, 2011). By calculating sensitivities directly with respect to the design variable, this method eliminates the computational overhead of finite difference methods.

More generally, shape calculus has been employed in the design of large-scale aerodynamic shape optimization problems such as complex aircraft components to produce exact gradient evaluations which allow iterative improvements of designs affordably (Schmidt *et al.*, 2011). Exact sensitivity derivatives, on the other hand, result in no computational overhead compared to finite difference approaches and improve design cycle turnaround.

2.3. Numerical methods and computational aerodynamics

In addition to pure analytical work, calculus based on real analysis provides the theoretical backbone for the numerical methods applied to solve the governing equations of aerodynamics. The finite difference, finite volume, and finite element methods are approximate techniques that enable the discretization of flow domains, lattices, or solving PDEs numerically (Moukalled *et al.*, 2016). The discrete form equations are established using integral calculus, and the boundary and initial conditions can be imposed as needed (Moukalled *et al.*, 2016).

Calculus-based memory aids are theories that have facilitated Computational Fluid Dynamics (CFD) by enabling more precise discretization and stability analyses of arithmetic methods. For example, involving time-fractional derivatives generalizes traditional differential calculus and leads to more realistic modeling of complex fluid mechanisms in turbulent/multi-dimensional flows. Aerospace professionals are interested in simulating realistic fluid behaviors under uncertainties, and this improved modelling is well aligned with these needs (Almheidat *et al.*, 2024).

Furthermore, aircraft control systems have been employing sophisticated mathematical tools such as the fractional-order calculus in order to mitigate that the inherent nonlinearities and non-stationarities associated with aerodynamic forces (Kopecny *et al.*, 2024). These groundbreaking applications subsume conventional flow modeling into a larger calculus space that aligns with the integration demands of flight control and system dynamics.

Aerodynamics is not performed in isolation; it has a direct connection with flight dynamics and control, as well as things that deal heavily with calculus. Tensor calculus and classical mechanics are the most commonly used tools for creating mathematical models of aircraft motion to represent translational and rotational kinematics and dynamics, especially based on Euler's equations of motion (Zipfel, 2014).

This concept plays a key role in designing flight control systems, helping achieve stability and efficiency by combining aerodynamic forces and moments through differential equations and linear algebra. Calculus integration allows for the derivation and solution of control laws, which are used to stabilize aircraft and deform trajectory paths against a variety of non-linear disturbances and aerodynamic interactions (Kaza & Kadari, 2025).

For instance, the employment of fractional-order lead compensators in aircraft control utilizes fractional calculus to provide greater resolution benefits over integer-order systems in the time and frequency domains, thus directly affecting the vehicle stability under aerodynamics fluctuations (Kopecny *et al.*, 2024).

2.4. Aerodynamic phenomena modeled by calculus

Calculus has a wide range of aerodynamic phenomena to model. These include boundary layer theory, shock wave formation in transonic and supersonic flows, turbulence modeling, and unsteady aerodynamics. Often simplified from the Navier–Stokes system, boundary layer equations, are used to make complex flow problems easier to study. By applying similarity transformations and symmetry analysis, these equations can be reduced to forms that allow analytical or semi-analytical solutions, offering practical insight into aerodynamic behavior near surfaces (Cantwell & Moulden, 2004).

Turbulence, a complex behavior in fluid dynamics, is represented using statistical and stochastic calculus to describe the fluctuation of velocity energy dissipation over many scales. A fully turbulent flow mathematical model is a far-off dream; however, in recent endeavors, calculus-based perturbations and scaling laws have shown potential for constituting usable practical turbulence models, notably aerospace (hypersonic flows) applications (Raje *et al.*, 2024).

Another example elucidating the beauty of calculus is its use in analyzing the classical problem of an elliptical lift distribution, which is related to minimum induced drag solutions and therefore efficient aircraft design: applications of calculus involving distributions of vortices and circulation perturbatively lead to low-drag solutions (MIT OpenCourseWare, n.d.).

2.5. Limitations and research gaps

Though its applications, calculus tools are highly known for their strength in the case of aerodynamics but still there is some difficulty. Exact analytical solutions for fluid flows are often unable to cope with the inherent nonlinearity and multiscale nature of these flows, giving rise to the need for numerical simulations, the accuracy of which is determined by discretization techniques and computational resources.

A fundamentally stochastic expression of the Navier-Stokes equation is turbulence that uses statistical and stochastic calculus to describe the effects of velocity fluctuations and energy dissipation over continuum scales. Although a complete mathematical model is still lacking, modern methods apply perturbations and scaling laws based on calculus to formulate feasible turbulence models for aerospace applications, specifically in the hypersonic sector (Raje *et al.*, 2024).

In addition, the theoretical and technological barriers that need to be cleared to achieve calculus-based optimization coupled with real-time flight control systems are a challenge for further cross-disciplinary research that can spearhead aerospace performance.

3. METHODOLOGY

3.1. Research question formulation and scope definition

The research started with a specific question for the investigation: "*How to use Calculus in aerodynamics at the major principles of this field, from state-of-the-art computational techniques to optimization frameworks employed in aerospace research and engineering practice?*"

In this case, the scope involved papers showing the use of calculus (e.g., differential equations, calculus of variations, numerical methods) in solving aerodynamic problems, aerospace design optimization, or flight dynamics modeling. This represents the wide variety of applications of calculus in aerodynamics, such as force models, simulations, and performance optimization.

3.2. Search strategy

A comprehensive search strategy was created to capture all relevant studies at the intersection of calculus and aerodynamics. The search strategy was comprehensive, incorporating a wide range of technical terms such as "*differential equations," "finite element," "airfoil," and "CFD.*" This way ensured that no important section was missed. Major databases were searched such as Scopus, Web of Science, AIAA ARC and electronic resource providers (IEEE Xplore, ScienceDirect, SpringerLink) and also in grey literature sources (technical reports, conference proceedings and dissertations). The whole approach kept in accordance with the PRISMA guidelines (Page *et al.*, 2021) for being systematic and transparent, documenting at all steps the flow of included and excluded study.

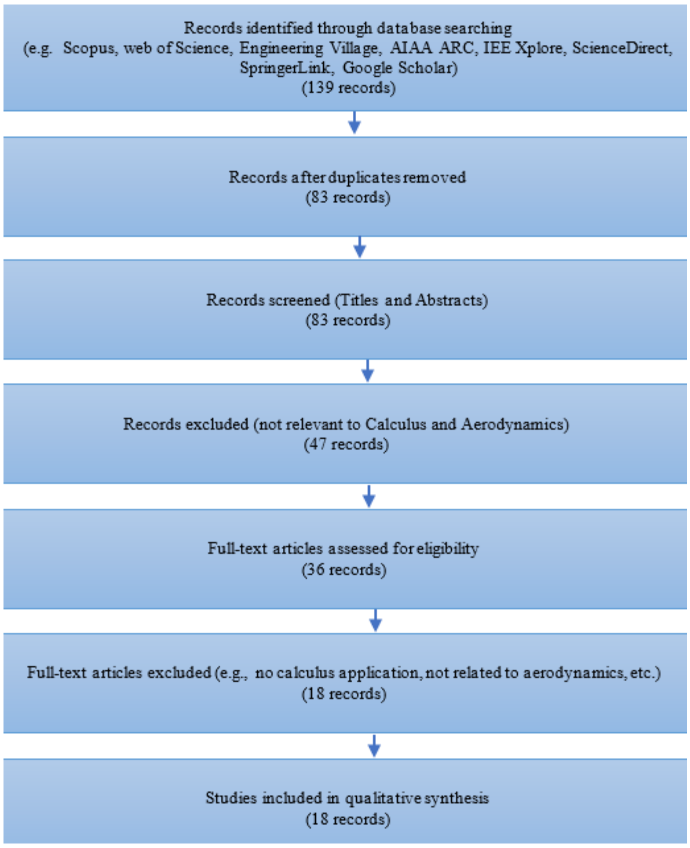

Figure 1. Comprehensive search strategy

3.3. Screening and eligibility criteria

At the screening stage, 83 records were retained after duplicates were removed. From this set, 47 were excluded following title and abstract review. The main reasons for exclusion included:

• Articles discussed aerodynamics without involving calculus or mathematical modeling (e.g., purely experimental wind tunnel studies).

• Publications that discussed mathematics or physics in general terms without establishing a clear link to aerodynamics.

• Non-scholarly or non-peer-reviewed sources (e.g., editorial notes, magazine articles, or brief conference abstracts with insufficient methodological detail).

This process resulted in 36 full-text articles assessed for eligibility. At the full-text review stage, an additional 18 studies were excluded. The reasons for exclusion at this stage were as follows:

• Papers that mentioned calculus but did not apply it within an aerodynamic context. (n = 7).

• Studies where calculus was applied only tangentially (e.g., in structural mechanics or thermodynamics) without addressing aerodynamic phenomena. (n = 6).

• Articles that lacked methodological rigor, such as incomplete reporting of equations, absence of reproducible procedures, or lack of peer review. (n = 3).

• Duplicate works that covered the same dataset or theory already represented in a more comprehensive source. (n = 2).

Ultimately, 18 studies met the inclusion criteria and were analyzed in this review.

3.4. Data extraction

A standard data extraction form was used in order to make the data collection procedures consistent. For every paper, the author(s), year of publication, source, type of calculus applied (where applicable; either integral calculus or differential calculus), and aerodynamic problem whether it was an airfoil design, controllability analysis, vehicle optimization, trajectory planning and guidance, turbulence modeling, drag reduction using active flow control device were all documented. This included the methodology used whether analytical, numerical or experimental as well as noted key findings what was accomplished and how calculus helped. One reviewer carried out data extraction, with the second reviewer conducting random cross-checks to reduce error and bias.

3.5. Synthesis of results

Findings were presented using narrative synthesis:

• Describing how calculus is applied in aerodynamic modeling

• Analyzing the effectiveness of different calculus-based techniques

• Highlighting variations across domains (e.g., CFD, optimization, control systems)

• Identifying research gaps and proposing directions for future work

3.6. Quality assessment

The quality assessment tools for simulations, theoretical models, and interventions were duly used to examine each study. The assessment used the following criteria:

• Rigor of methodology

• Validity of conclusions

• Clarity and transparency of reporting

This evaluation helped assess the strength of the evidence and reduce the risk of bias.

4. RESULTS AND DISCUSSION

4.1. Overview of included studies

Eighteen studies were ultimately included in the review and synthesized. A review of the studies selected shows the foundations and the new frontiers of aerodynamics. Earlier works like Anderson and White provided theories. More recent studies, for example, Chai *et al.* (2023) and Almheidat *et al.* (2024) highlight advances in computational modeling and optimization. This review emphasizes four key aspects. The first one is modeling aerodynamic behavior using differential equations. The second focuses on applying variational and shape calculus for optimization. The third emphasizes the use of numerical methods in computational fluid dynamics. Finally, calculus is playing an increasingly important role in flight dynamics and control.

Table 1. Summary of 18 studies of calculus in aerodynamics

Author/Year	Type of Calculus	Aerodynamic Focus	Methodology	Key Finding
Anderson (2011)	Differential Equations	Fluid flow & aerodynamics	Analytical & numerical	Navier–Stokes and Euler equations remain foundational; limited analytical solutions, reliance on numerical methods
White (2016)	Differential Equations	Fluid mechanics	Analytical	Fluid mechanics formulations rely on PDEs for flow representation
White & Xue (2021)	Differential Equations	Navier–Stokes applications	Analytical & numerical	Nonlinear PDEs essential in turbulence/ laminar flow, solved mainly with numerical approaches
Almheidat *et al.* (2024)	Fractional Calculus	Turbulence modeling	Numerical (time-fractional Navier–Stokes)	Fractional derivatives improve modeling of turbulence and multidimensional flow systems
Sahani *et al.* (2023)	Differential Calculus	Aircraft geometry & flow fields	Numerical methods	Differential calculus essential for modeling aerodynamic processes and solving nonlinear problems
Kaza & Kadari (2025)	Vector Calculus	Lift, drag, circulation	Analytical	Gradient, divergence, and curl operators explain vorticity and circulation central to lift/drag forces

Sadrehaghighi (2023)	Differential Equations	Basic aerodynamics	Educational primer	Summarizes lift, drag, and flow regimes through PDEs
Ibrahim & Tiwari (2004)	Calculus of Variations	Design optimization	Variational method	Used to optimize aerodynamic shapes and trajectories with sensitivity analysis
Gallant (2012)	Calculus of Variations	Flight profile optimization	Variational method	Identified optimal climb, cruise, and descent profiles considering efficiency and constraints
Schmidt *et al.* (2011)	Shape Calculus	Airfoil/wing optimization	Adjoint-based optimization	Shape calculus yields accurate gradients for iterative aerodynamic design improvements
Chai *et al.* (2023)	Calculus of Variations	Trajectory optimization	Variational + numerical	Developed advanced trajectory optimization under nonlinear and operational constraints
Moukalled *et al.* (2016)	Integral & Differential Calculus	CFD numerical methods	Finite volume method	Integral calculus underpins discretization and boundary condition application in CFD
Cantwell & Moulden (2004)	Differential Calculus	Boundary layer theory	Analytical	Similarity transformations in boundary layer equations provide semi-analytical solutions
Raje *et al.* (2024)	Stochastic/ Statistical Calculus	Turbulence modeling	Perturbation & scaling laws	Applied statistical calculus to turbulence in hypersonic flow regimes
Kopecny *et al.* (2024)	Fractional Calculus	Flight control systems	Fractional-order compensators	Fractional-order control improves robustness and stability under nonlinear aerodynamic forces
Zipfel (2014)	Differential & Integral Calculus	Flight dynamics/ control	Modeling & simulation	Euler's equations of motion modeled with calculus; improved stability and trajectory control
MIT OCW (n.d.)	Differential Calculus	Induced drag modeling	Analytical	Elliptical lift distribution minimizes induced drag using vortex and circulation theory
NASA (2024)	Differential Equations	Navier–Stokes explanations	Educational/ technical report	Explains fundamental role of Navier–Stokes equations in aerodynamics

4.2. Differential equations in aerodynamic modeling

Seven studies emphasized the role of differential equations, particularly the Navier–Stokes and Euler formulations, as the cornerstone of aerodynamics (Anderson, 2011; White, 2016; White & Xue, 2021; Sadrehaghighi, 2023; Cantwell & Moulden, 2004; NASA, 2024; Sahani *et al.*, 2023). Overall, these findings emphasize that partial differential equations are essential, yet their true power can only be harnessed with computational support (Table 1).

4.3. Calculus of variations and shape calculus in optimization

Five studies applied variational methods and shape calculus to improve aerodynamic efficiency (Ibrahim & Tiwari, 2004; Gallant, 2012; Schmidt *et al.*, 2011; Chai *et al.*, 2023; Kaza & Kadari, 2025). These works show how calculus can refine airfoil shapes, optimize flight trajectories, and reduce drag. For example, Gallant (2012) and Chai *et al.* (2023) demonstrated the use of calculus of variations for trajectory optimization, while Schmidt *et al.* (2011) used shape calculus to enable iterative design improvements through gradient-based methods. Together, these studies highlight the consistent value of optimization frameworks in linking mathematical theory to engineering outcomes (Table 1).

4.4. Numerical methods and computational fluid dynamics

Four studies (Moukalled *et al.*, 2016; Almheidat *et al.*, 2024; Sahani *et al.*, 2023; Raje *et al.*, 2024) focused on the integration of calculus into CFD. These included discretization approaches based on integral and differential calculus, such as finite volume and finite element methods (Moukalled *et al.*, 2016), and the use of fractional derivatives to enhance turbulence modeling (Almheidat *et al.*, 2024). Raje *et al.* (2024) extended this line by applying stochastic calculus for turbulence in hypersonic flows. Collectively, these studies confirm that calculus is the theoretical backbone of CFD, while also revealing constraints related to computational cost and resolution (Table 1).

4.5. Integration with flight dynamics and control systems

Two studies have investigated how calculus is applied in flight dynamics and control (Zipfel, 2014; Kopecny *et al.*, 2024). Zipfel (2014) employed Euler's equations of motion to extend aircraft stability models, whereas Kopecny and others (2024) utilized fractional-order control laws for enhanced robustness under nonlinear aerodynamic conditions. While still restricted, these studies represent a developing research area where calculus is explicitly applied to strategies for stability and control (Table 1).

4.6 Critical synthesis

This synthesis shows that calculus functions at multiple levels: as a theoretical foundation (Anderson, 2011; White & Xue, 2021), a design optimization tool (Schmidt *et al.*, 2011; Chai *et al.*, 2023), a computational enabler (Moukalled *et al.*, 2016; Almheidat *et al.*, 2024), and an operational stabilizer (Zipfel, 2014; Kopecny *et al.*, 2024). While the evidence base confirms the indispensable role of calculus in aerodynamics, it also reveals several gaps: (1) limited exploration of real-time applications in autonomous aerospace systems, (2) a lack of integrated frameworks combining PDE-based models with machine learning approaches, and (3) underrepresentation of fractional calculus in applied aerodynamics despite promising initial results.

Examining these themes shows that calculus isn't merely a theoretical tool, it has a hands-on role in advancing innovation in aerodynamics. At the same time, the evidence is scattered, which points to the need for research that combines theory, computational methods, and hands-on applications

5. CONCLUSION

This systematic review united the evidence of 18 studies regarding the use of calculus in aerodynamics. The results indicate that calculus works in four thematic domains, which are (1) differential equations to model aerodynamics (7 studies; 39%), (2) calculus of variations and shape calculus to optimize (5 studies; 28%), (3) numerical methods as computational fluid dynamics (CFD) (4 studies; 22%), (4) integration with flight dynamics and control (2 studies; 11%).

Differential equations remain the theoretical foundation, underpinning the Navier–Stokes and Euler formulations essential for describing aerodynamic flows. Optimization approaches such as variational and shape calculus act as design tools, consistently improving wing geometries, flight trajectories, and overall efficiency. At present time, the application of numerical methods, whether based on integral calculus or differential calculus, serves as useful computational helps, allowing some PDEs to become solvable by discretization and fractional formulation, for example. Finally, the new control applications of fractional calculus reveal its operational stabilizer however this domain is still under researched.

Despite these advances, important gaps remain. Not many studies focused on real time application of calculus-based optimization techniques. PDE based models are poorly integrated with machine learning techniques which could relax computational bottleneck. Overall, fractional calculus is probably more applicable for turbulence modeling or control systems than used in applied aerodynamics.

Overall, this review confirms that calculus is not only a theoretical necessity but also a practical driver of innovation in aerodynamics. Future research must move toward interdisciplinary approaches that bridge mathematical theory, computational power, and applied engineering in order to fully realize the potential of calculus in next-generation aerospace systems.

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